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**REFERENCES**

1. N. L. Alling, Foundations of Analysis on Surreal Number Fields, North-Holland Publishing Co., 1987.
2. E. Hewitt, Rings of continuous functions I, Trans. Amer. Math. Soc. 64(1948), 54-99.

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# MINIMAL STRUCTURES, *m*-OPEN MULTIFUNCTIONS AND BITOPOLOGICAL SPACES

TAKASHI NOIRI AND VALERIU POPA

**ABSTRACT :** By using *m*-open multifunctions from a topological space into an *m*-space, we establish the unified theory for several weak forms of open multifunctions between bitopological spaces.

**Key words and pharases :** *m*-structure, *m*-open set, $(i, j)$-*m*-open multifunction, bitopological space, multifunction.

**AMS Subject Classification : 54A05, 54C10, 54C60, 54E55.**

## 1. INTRODUCTION

Semi-open sets, preopen sets, α-open sets and β-open sets play an important role in the researching of generalizations of open functions and open multifunctions. By using these sets, several authors introduced and studied various types of modifications of open functions and open multifunctions in topological spaces and bitopological spaces. Maheshwari and Prasad [20] and Bose [6] introduced the concepts of semi-open sets and semi-open functions in bitopological spaces.. Jelic' [12], [14], Kar and Bhattacharyya [15] and Khedr et al. [16] introduced and studied the concepts of preopen sets and preopen functions in bitopological spaces. The notions of α-open sets and α-open functions in bitopological spaces were studied in [13], [24] and [17]. Some forms of open multifunctions are studied in [5], [7] and [8]. Recently, in [30] and [31] the present authors introduced the notions of minimal structures, *m*-spaces and *m*-continuity.

In the present paper, we introduce the notion of an *m*-open multifunctions from a topological space into an *m*-space and establish the unified theory for several weak forms of open multifunctions between bitopological spaces. We obtain some characterizations of *m*-open multifunctions and characterize the set of all points at which a multifunction is not *m*-open. In the last part, some new modifications of open multifunctions between bitopological spaces is introduced and investigated.

## 2. PRELIMINARIES

Let $(X, \tau)$ be a topological space and $A$ a subset of $X$. The closure of $A$ and the interior of $A$ are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively.

**Definition 2.1.** Let $(X, \tau)$ be a topological space. A subset $A$ of $X$ is said to be *α-open* [25] (resp. *semi-open* [18], *preopen* [22], *β-open* [1] or *semi-preopen* [3], *b-open* [4] or *γ-open* [11]) if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$ (resp. $A \subset \mathrm{Cl}(\mathrm{Int}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(A))$, $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$, $A \subset \mathrm{Int}(\mathrm{Cl}(A)) \cup \mathrm{Cl}(\mathrm{Int}(A))$).

The family of all semi-open (resp. preopen, α-open, β-open, semi-preopen, *b*-open) sets in $X$ is denoted by $SO(X)$ (resp. $PO(X)$, $\alpha(X)$, $\beta(X)$, $SPO(X)$, $BO(X)$).

**Definition 2.2.** The complement of a semi-open (resp. preopen, α-open, β-open, semi-preopen, *b*-open) set is said to be *semi-closed* [9] (resp. *preclosed* [10], *α-closed* [23], *β-closed* [1], *semi-preclosed* [3], *b-closed* [4]).

**Definition 2.3.** The intersection of all semi-closed (resp. preclosed, α-closed, β-closed, semi-preclosed, *b*-closed) sets of $X$ containing $A$ is called the *semi-closure* [9] (resp. *pre-closure* [10], *α-closure* [23], *β-closure* [2], *semi-preclosure* [3], *b-closure* [4]) of $A$ and is denoted by $s\mathrm{Cl}(A)$ (resp. $p\mathrm{Cl}(A)$, $\alpha\mathrm{Cl}(A)$, ${}_{\beta}\mathrm{Cl}(A)$, $sp\mathrm{Cl}(A)$, $b\mathrm{Cl}(A)$).

**Definition 2.4.** The union of all semi-open (resp. prepen, α-open, β-open, semi-preopen, *b*-open) sets of $X$ contained in $A$ is called the *semi-interior* (resp. *preinterior, α-interior, β-interior, semi-preinterior, b-interior*) of $A$ and is denoted by $s\mathrm{Int}(A)$ (resp. $p\mathrm{Int}(A)$, $\alpha\mathrm{Int}(A)$, $\beta\mathrm{Int}(A)$, $sp\mathrm{Int}(A)$, $b\mathrm{Int}(A)$).

Throught the present paper, $(X, \tau)$ and $(Y, \sigma)$ (briefly $X$ and $Y$) always denote topological spaces and $F : X \to Y$ (resp. $f : X \to Y$) presents a multivalued (resp. single valued) function. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a subset $B$ of a space $Y$ by $F^+(B)$ and $F^-(B)$, respectively, that is

$$F^+(B) = \{x \in X : F(x) \subset B\} \text{ and } F^-(B) = \{x \in X : F(x) \cap B \neq \emptyset\}.$$

**Definition 2.5.** A function $f : (X, \tau) \to (Y, \tau)$ is said to be *open* at a point $x \in X$ if for each open set $U$ containing $x$, there exists an open set $V$ of $Y$ containing $f(x)$ such that $V \subset f(U)$. If $f$ is open at each point $x \in X$, then $f$ is said to be *open*.

**Remark 2.1.** A function $f : (X, \tau) \to (Y, \sigma)$ is open if and only if $f(U)$ is open for each open set $U$ of $X$.

**Definition 2.6.** function $f : (X, \tau) \to (Y, \sigma)$ is said to be *semi-open* [26] (resp. *preopen* [22], *α-open* [23], *β-open* [1]) if $f(U)$ is semi-open (resp. preopen, α-open, β-open) for each open set $U$ of $X$.

**Definition 2.7.** A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be *open* [5] (resp. *semi-open* [29], *preopen* [8], *α-open* [7], *β-open*) if $F(U)$ is open (resp. semi-open, preopen, α-open, β-open) for each open set $U$ of $X$.

## 3. MINIMAL STRUCTURES AND *m*-OPEN MULTIFUNCITONS

**Definition 3.1.** A subfamily $m_X$ of the power set $\mathcal{P}(X)$ of a nonempty set $X$ is called a *minimal structure* (or briefly *m-structure*) [30], [31] on $X$ if $\emptyset \in m_X$ and $X \in m_X$.

By $(X, m_X)$ (or briefly $(X, m)$), we denote a nonempty set $X$ with a minimal structure $m_X$ on $X$ and call it an *m-space*. Each member of $m_X$ is said to be *$m_X$-open* (or briefly *m-open*) and the complement of an $m_X$-open set is said to be *$m_X$-closed* (or briefly *m-closed*).

**Definition 3.2.** Let $X$ be a nonempty set and $m_X$ an $m$-structure on $X$. For a subset $A$ of $X$, the *$m_X$-closure* of $A$ and the *$m_X$-interior* of $A$ are defined in [21] as follows:

(1) $m_X\text{-Cl}(A) = \cap\{F : A \subset F, X - F \in m_X\}$,

(2) $m_X\text{-Int}(A) = \cup\{U : U \subset A, U \in m_X\}$.

**Remark 3.1.** Let $(X, \tau)$ be a topological space and $A$ be a subset of $X$. If $m_X = \tau$ (resp. $SO(X)$, $PO(X)$, $\alpha(X)$, $\beta(X)$, $BO(X)$), then we have

(a) $m_X\text{-Cl}(A) = \text{Cl}(A)$ (resp. $\text{sCl}(A)$, $\text{pCl}(A)$, $\alpha\text{Cl}(A$, $\beta\text{Cl}(A)$, $\text{bCl}(A)$),

(b) $m_X\text{-Int}(A) = \text{Int}(A)$ (resp. $\text{sInt}(A)$, $\text{pInt}(A)$, $\alpha\text{Int}(A$, $\beta\text{Int}(A)$, $\text{bInt}(A)$).

**Lemma 3.1.** (Maki et al. [21]). *Let $(X, m_X)$ be an m-space. For subsets $A$ and $B$ of $X$, the following properties hold:*

(1) $m_X\text{-Cl}(X - A) = X - m_X\text{-Int}(A)$ *and* $m_X\text{-Int}(X - A) = X - m_X\text{-Cl}(A)$,

(2) *If* $(X - A) \in m_X$, *then* $m_X\text{-Cl}(A) = A$ *and if* $A \in m_X$, *then* $m_X\text{-Int}(A) = A$,

(3) $m_X\text{-Cl}(\emptyset) = \emptyset$, $m_X\text{-Cl}(X) = X$, $m_X\text{-Int}(\emptyset) = \emptyset$ *and* $m_X\text{-Int}(X) = X$,

(4) *If* $A \subset B$, *then* $m_X\text{-Cl}(A) \subset m_X\text{-Cl}(B)$ *and* $m_X\text{-Int}(A) \subset m_X\text{-Int}(B)$,

(5) $A \subset m_X\text{-Cl}(A)$ *and* $m_X\text{-Int}(A) \subset A$,

(6) $m_X\text{-Cl}(m_X\text{-Cl}(A)) = m_X\text{-Cl}(A)$ *and* $m_X\text{-Int}(m_X\text{-Int}(A)) = m_X\text{-Int}(A)$.

**Lemma 3.2.** (Popa and Noiri [30]). *Let $(X, m_X)$ be an m-space and $A$ a subset of $X$. Then $x \in m_X\text{-Cl}(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing $x$.*

**Definition 3.3.** A minimal structure $m_X$ on a nonempty set $X$ is said to have *property* $\mathcal{B}$ [21] if the union of any family of subsets belonging to $m_X$ belongs to $m_X$.

**Definition 3.3.** (Popa and Noiri [32]). *Let $(X, m_X)$ be an m-space and $m_X$ satisfy property $\mathcal{B}$. Then for a subset $A$ of $X$, the following properties hold:*

(1) $A \in m_X$ *if and only if* $m_X\text{-Int}(A) = A$,

(2) *$A$ is m-closed if and only if* $m_X\text{-Cl}(A) = A$,

(3) $m_X$-Int$(A) \in m_X$ *and* $m_X$-Cl$(A)$ *is* $m_X$*-closed.*

**Remark 3.2.** Let $(X, \tau)$ be a topological space and $m_X$ = SO$(X)$ (resp. PO$(X)$, $\alpha(X)$, $\beta(X)$, BO$(X)$), then $m_X$ satisfies property $\mathcal{B}$.

**Definition 3.4.** Let $(Y, m_Y)$ be an $m$-space.

(1) A multifunction $F : (X, \tau) \to (Y, m_Y)$ is said to be *m-open* at $x \in X$ if for each open set $U$ containing $x$, there exists $V \in m_Y$ containing $F(x)$ such that $V \subset F(U)$. If $F$ is $m$-open at each point $x \in X$, then $F$ is said to be *m-open*.

(2) A function $f : (X \tau) \to (Y, m_Y)$ is said to be *m-open* at $x \in X$ if for each open set $U$ containing $x$, there exists $V \in m_Y$ containing $f(x)$ such that $V \subset f(U)$. If $f$ is $m$-open at each point $x \in X$, then $f$ is said to be *m-open*.

**Theorem 3.1.** *A multifunction* $F : (X, \tau) \to (Y, m_Y)$ *is m-open at* $x \in X$*, where* $m_Y$ *has property* $\mathcal{B}$*, if and only if for each open set U containing, x,* $x \in F^+(m_Y\text{-Int}(F(U)))$.

**Proof.** *Necessity.* Let $U$ be an open set containing $x$. Then, there exists $V \in m_Y$ such that $F(x) \subset V \subset F(U)$ and hence $F(x) \subset m_Y$-Int$(F(U))$. Therefore, we obtain that $x \in F^+(m_Y\text{-Int}(F(U)))$.

*Sufficiency.* Suppose that $x \in F^+(m_Y\text{-Int}(F(U)))$ for each open set U containing $x$. Then $F(x) \subset m_Y$-Int$(F(U))$. Set $V = m_Y$-Int$(F(U))$, then by Lemma 3.3 $V \in m_Y$ and $F(x) \subset V \subset F(U)$. Therefore, $F$ is $m$-open at $x$.

**Theorem 3.2.** *A multifunction* $F : (X, \tau) \to (Y, m_Y)$ *is m-open, where* $(Y, m_Y)$ *has property B, if and only if F(U) is* $m_Y$*-open for each open set U of X.*

**Proof.** *Necessity.* Let $U$ be an open set of $X$ and $x \in U$. Since $F$ is $m$-open at $x \in X$, by Theorem 3.1 we have $F(x) \subset m_Y$-Int$(F(U))$ and by Lemma 3.1 $F(U) = m_Y$-Int$(F(U))$. By Lemma 3.3, $F(U)$ is $m_Y$-open.

*Sufficiency.* Let $x \in X$ and $U$ be an open set of $X$ containing $x$. Then we have $F(x) \subset F(U) = m_Y$-Int$(F(U))$. Therefore $x \in F^+(m_Y\text{-Int}(F(U))$. By Theorem 3.1, $F$ is $m$-open at arbitrary point $x \in X$.

**Remark 3.3.** (a) If $F : (X, \tau) \to (Y, \sigma)$ is a multifunciton and $m_Y = \tau$ (resp. SO$(Y)$, PO$(Y)$, $\alpha(Y)$, $\beta(Y)$), we obtain Definition 2.7, that is, the definition of an open (resp. semi-open, preopen, $\alpha$-open, $\beta$-open) multifunction, where SO$(Y)$ (resp. PO$(Y)$, $\alpha(Y)$, $\beta(Y)$) is the family of all semi-open (resp. preopen, $\alpha$-open, $\beta$-open) sets of $Y$.

(b) If $f : (X, \tau) \to (Y, \sigma)$ is a function and $m_Y = \sigma$ (resp. SO$(Y)$, PO$(Y)$, $\alpha(Y)$, $\beta(Y)$), we obtain Definition 2.6.

**Theorem 3.3.** *For a multifunction* $F : (X, \tau) \to (Y, m_Y)$*, where* $m_Y$ *has property* $\mathcal{B}$*, the following properties are equivalent:*

(1) *$F$ is m-open at $x$;*

(2) *If $x \in \mathrm{Int}(A)$ for $A \in \mathcal{P}(X)$, then $x \in F^+(m_Y\text{-}\mathrm{Int}(F(A)))$;*

(3) *$x \in \mathrm{Int}(F^+(B))$ for $B \in \mathcal{P}(Y)$, then $x \in F^+(m_Y\text{-}\mathrm{Int}(B))$;*

(4) *If $x \in F^-(m_Y\text{-}\mathrm{Cl}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in \mathrm{Cl}(F^-(B))$.*

**Proof.** (1) $\Rightarrow$ (2) : Let $A \in \mathcal{P}(X)$ and $x \in \mathrm{Int}(A)$. Then, there exists an open set $U$ such that $x \in U \subset A$ and hence $F(x) \subset F(U) \subset F(A)$. Since $F$ is $m$-open at $x$, by Theorem 3.1 and Lemma 3.1, we obtain $x \in F^+(m_Y\text{-}\mathrm{Int}(F(U))) \subset F^+(m_Y\text{-}\mathrm{Int}(F(A))))$.

(2) $\Rightarrow$ (3) : Let $B \in \mathcal{P}(Y)$ and $x \in \mathrm{Int}\ (\mathrm{F}^+(B))$. Then, $x \in F^+(m_Y\text{-}\mathrm{Int}(F(F^+(B)))) \subset F^+(m_Y\text{-}\mathrm{Int}(B))$.

(3) $\Rightarrow$ (4) : Let $B \in \mathcal{P}(Y)$ and $x \notin \mathrm{Cl}(F^-(B))$. Then $x \in X - \mathrm{Cl}(F^-(B)) = \mathrm{Int}(X - F^-(B)) = \mathrm{Int}(F^+(Y - B))$. By (3) we have $x \in F^+(m_Y\text{-}\mathrm{Int}(Y-B)) = X - F^-(m_Y\text{-}\mathrm{Cl}(B))$. Hence, $x \notin F^-(m_Y\text{-}\mathrm{Cl}(B))$. Therefore, if $x \in F^-(m_Y\text{-}\mathrm{Cl}(B))$, then $x \in \mathrm{Cl}(F^-(B))$.

(4) $\Rightarrow$ (1) : Let $U$ be any open set of $X$ containing $x$ and $B = Y - F(U)$. Since $\mathrm{Cl}(F^-(B)) = \mathrm{Cl}(F^-(Y - F(U))) = \mathrm{Cl}(X - F^+(F(U))) \subset X - \mathrm{Int}(U) = X - U$ and $x \in U$, we obtain that $x \notin \mathrm{Cl}(F^-(B))$. By (4), we have $x \notin F^-(m_Y\text{-}\mathrm{Cl}(B)) = F^-(m_Y\text{-}\mathrm{Cl}(Y - F(U))) = X - F^+(m_Y\text{-}\mathrm{Int}(F(U)))$. Therefore, $x \in F^+(m_Y\text{-}\mathrm{Int}(F(U)))$. By Theorem 3.1, $F$ is $m$-open at $x$.

**Theorem 3.4.** *For a multifunciton $F : (X, \tau) \to (Y, m_Y)$, where $m_Y$ has property $\mathcal{B}$, the following properties are equivalent:*

(1) *$F$ is m-open;*

(2) *$F(\mathrm{Int}(A)) \subset m_Y\text{-}\mathrm{Int}(F(A))$ for any subset $A$ of $X$;*

(3) *$\mathrm{Int}(F^+(B)) \subset F^+(m_Y\text{-}\mathrm{Int}(B))$ for any subset $B$ of $Y$;*

(4) *$F^-(m_Y\text{-}\mathrm{Cl}(B)) \subset \mathrm{Cl}(F^-(B))$ for any subset $B$ of $Y$.*

**Proof.** (1) $\Rightarrow$ (2) : Let $A$ be any subset of $X$ and $x \in \mathrm{Int}(A)$. Since $F$ is $m$-open at each $x \in A$, by Theorem 3.3 $F(x) \subset m_Y\text{-}\mathrm{Int}(F(A)$. Hence $F(\mathrm{Int}(A)) \subset m_Y\text{-}\mathrm{Int}(F(A))$.

(2) $\Rightarrow$ (3): Let $B$ be any subset of $Y$. By (2), we have $F(\mathrm{Int}(F^+(B))) \subset m_Y\text{-}\mathrm{Int}(F(F^+(B))) \subset m_Y\text{-}\mathrm{Int}(B)$.

(3) $\Rightarrow$ (4): Let $B$ be any subset of $Y$. By (3), we have $X - \mathrm{Cl}(F^-(B)) = \mathrm{Int}(X - F^-(B)) = \mathrm{Int}(F^+(Y - B)) \subset F^+(m_Y\text{-}\mathrm{Int}(Y - B)) = X - F^-(m_Y\text{-}\mathrm{Cl}(B))$. Hence, $F^-(m_Y\text{-}\mathrm{Cl}(B)) \subset \mathrm{Cl}(F^-(B))$.

(4) $\Rightarrow$ (1): Let $U$ be any open set of $X$ and $B = Y - F(U)$. By (4), we have $F^-(m_Y\text{-}\mathrm{Cl}(Y - F(U))) \subset \mathrm{Cl}(F^-(Y - F(U))$. Now, $F^-(m_Y\text{-}\mathrm{Cl}(Y - F(U))) = F^-(Y - m_Y\text{-}\mathrm{Int}(F(U))) = X - F^+(m_Y\text{-}\mathrm{Int}(F(U)))$. And also we have $\mathrm{Cl}(F^-(Y - F(U))) = \mathrm{Cl}(X - F^+(F(U))) \subset X - \mathrm{Int}(U) = X - U$. Therefore, we obtain $U \subset F^+(m_Y\text{-}\mathrm{Int}(F(U)))$ and hence $F(U) \subset m_Y\text{-}\mathrm{Int}\ (F(U))$. By Lemma 3.1, $F(U) = m_Y\text{-}\mathrm{Int}(F(U)$ and by Theorem 3.2, $F$ is $m$-open.

**Remark 3.4.** *Let* $F : (X, \tau) \to (Y, \sigma)$ be a multifunction and $m_Y = \mathrm{SO}(X)$. Then by Theorem 3.4, we obtain the result established in Lemma 5.2 of [29].

For a multifunction $F : (X, \sigma) \to (Y, m_Y)$, we denote

$$D^0(F) = \{x \in X : F \text{ is not } m\text{-open at } x\}.$$

**Theorem 3.5.** *For a multifunction* $F : (X, \tau) \to (Y, m_Y)$, *where* $m_Y$ *has property* $\mathcal{B}$, *the following properties hold:*

$$\begin{aligned} D^0(F) &= \cup_{U \in \tau}\{U - F^-(m_Y\text{-Int}(F(U)))\} \\ &= \cup_{A \in \mathcal{P}(X)}\{\mathrm{Int}(A) - F^+(m_Y\text{-Int}(F(A)))\} \\ &= \cup_{B \in \mathcal{P}(Y)}\{\mathrm{Int}(F^+(B)) - F^+(m_Y\text{-Int}(B))\} \\ &= \cup_{B \in \mathcal{P}(Y)}\{F^-(m_Y\text{-Cl}(B) - \mathrm{Cl}(F^-(B))\}. \end{aligned}$$

**Proof.** Let $x \in D^0(F)$. Then, by Theorem 3.1, there exists an open set $U_0$ containing $x$ such that $x \notin F^+(m_Y\text{-Int}(F(U_0)))$. Hence $x \in U_0 \cap (X \setminus F^+(m_Y\text{-Int}(F(U_0)))) = U_0 \setminus F^+(m_Y\text{-Int}(F(U_0)))$ $\subset \cup_{U \in \tau}\{U - F^+(m_Y\text{-Int}(F(U)))\}$.

Conversely, let $x \in \cup_{U \in \tau}\{U - F^+(m_Y\text{-Int}(F(U)))\}$. Then there exists $U_0 \in \tau$ such that $x \in U_0 - F^+(m_Y\text{-Int}(F(U_0)))$. Therefore, by Theorem 3.1 $x \in D^0(F)$.

For the second equation, let $x \in D^0(F)$. Then, by Theorem 3.3, there exists $A_1 \in \mathcal{P}(X)$ such that $x \in \mathrm{Int}(A_1)$ and $x \notin F^+(m_Y\text{-Int}(F(A_1)))$. Therefore, $x \in \mathrm{Int}(A_1) - F^+(m_Y\text{-Int}(F(A_1)))$ $\subset \cup_{A \in \mathcal{P}(x)}\{\mathrm{Int}(A) - F^+(m_Y\text{-Int}(F(A)))\}$.

Conversely, $x \in \cup_{A \in \mathcal{P}(x)}\{\mathrm{Int}(A) - F^+(m_Y\text{-Int}(F(A)))\}$. Then there exists $A_1 \in \mathcal{P}(X)$ such that $x \in \mathrm{Int}(A_1) - F^+(m_Y\text{-Int}(F(A_1)))$. By Theorem 3.3, $x \in D^0(f)$.

The other equations are similarly proved.

## 4. MINIMAL STRUCTURES AND BITOPOLOGICAL SPACES

Throughout the present paper, $(X, \tau_1, \tau_2)$ and $(Y, \sigma_1, \sigma_2)$ denote bitopological spaces. For a subset $A$ of $X$, the closure of $A$ and the interior of $A$ with respect to $\tau_i$ are denoted by $i\mathrm{Cl}(A)$ and $i\mathrm{Int}(A)$, respectively, for $i = 1, 2$. First, we shall recall some definitions of weak forms of open sets in a bitopological space.

**Definition 4.1.** A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is said to be

(1) *$(i, j)$-semi-open* [20] if $A \subset j\mathrm{Cl}(i\mathrm{Int}(A))$, where $i \neq j$, $i, j = 1, 2$,

(2) *$(i, j)$-preopen* [12] if $A \subset i\mathrm{Int}(j\mathrm{Cl}(A))$, where $i \neq j$, $i, j = 1, 2$,

(3) *$(i, j)$-$\alpha$-open* [13] if $A \subset i\mathrm{Int}(j\mathrm{Cl}(i\mathrm{Int}(A)))$, where $i \neq j$, $i, j = 1, 2$,

(4) *$(i, j)$-semi-preopen* [16] if there exists an $(i, j)$-preopen set $U$ such that $U \subset A \subset j\mathrm{Cl}(U)$, where $i \neq j$, $i, j = 1, 2$.

The family of $(i, j)$-semi-open (resp. $(i, j)$-preopen, $(i, j)$-$\alpha$-open $(i, j)$-semi-preopen) sets of $(X, \tau_1, \tau_2)$ is denoted by $(i, j)SO(X)$ (resp. $(i, j)PO(X)$, $(i, j)\alpha(X)$, $(i, j)SPO(X)$).

**Remark 4.1.** Let $(X, \tau_1, \tau_2)$ be a bitopological space and $A$ a subset of $X$. Then $(i, j)SO(X)$, $(i, j)PO(X)$, $(i, j)\alpha(X)$ and $(i, j)SPO(X)$ are all $m$-structures on $X$. Hence, if $m_{ij} = (i, j)SO(X)$ (resp. $(i, j)PO(X)$, $(i, j)\alpha(X)$, $(i, j)SPO(X)$), then we have

(1) $m_{ij}$-$\mathrm{Cl}(A) = (i, j)$-$s\mathrm{Cl}(A)$ [20] (resp. $(i, j)$-$\mathrm{pCl}(A)$ [16], $(i, j)$-$\alpha\mathrm{Cl}(A)$ [24], $(i, j)$-$\mathrm{spCl}(A)$ [16]),

(2) $m_{ij}$-$\mathrm{Int}(A) = (i, j)$-$\mathrm{sInt}(A)$ (resp. $(i, j)$-$\mathrm{pInt}(A)$, $(i, j)$-$\alpha\mathrm{Int}(A)$, $(i, j)$-$\mathrm{spInt}(A)$).

**Remark 4.2.** Let $(X, \tau_1, \tau_2)$ be a bitopological space.

(a) Let $m_i = (i, j)SO(X)$ (resp. $(i, j)\alpha(X)$). Then, by Lemma 3.1 we obtain the result established in Theorem 13 of [20] and Theorem 1.13 of [19] (resp. Theorem 3.6 of [24]).

(b) Let $m_{ij} = (i, j)SO(X)$ (resp. $(i, j)PO(X)$, $(i, j)\alpha(X)$, $(i, j)SPO(X)$). Then, by Lemma 3.2 we obtain the result established in Theorem 1.15 of [19] (resp. Theorem 3.5 of [16], Theorem 3.5 of [24], Theorem 3.6 of [16]).

**Remark 4.3.** Let $(X, \tau_1, \tau_2)$ be a bitopological space.

(a) It follows from Theorem 2 of [20] (resp. Theorem 4.2 of [15] or theorem 3.2 of [16], Theorem 3.2 of [24], Theorem 3.2 of [16] that $(i, j)SO(X)$ (resp. $(i, j)PO(X)$, $(i, j)\alpha(X)$, $(i, j)SPO(X)$) is an $m$-structure on $X$ satisfying property $\mathcal{B}$.

(b) Let $m_{ij} = (i, j)SO(X)$ (resp. $(i, j)PO(X)$, $(i, j)\alpha(X)$, $(i, j)SPO(X)$). Then, by Lemma 3.3 we obtain the result established in Theorem 1.13 of [19] (resp. Theorem 3.5 of [16], Theorem 3.6 of [24], Theorem 3.6 of [16]).

## 5. $m$-OPEN MULTIFUNCTIONS IN BITOPOLOGICAL SPACES

**Definition 5.1.** A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be *$(i, j)$-semi-open* [6] (resp. *$(i, j)$-preopen* [15], *$(i, j)$-$\alpha$-open* [17], *$(i, j)$-semi-preopen*) if for each $\tau_i$-open set $U$ of $X$, $f(U)$ is $(i, j)$-semi-open (resp. $(i, j)$-preopen, $(i, j)$-$\alpha$-open, $(i, j)$-semi-prepen) in $Y$.

**Definition 5.2.** A multifunction $F : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be *$(i, j)$-almost open* (or *$(i, j)$-preopen*) [8] if for each $U \in \tau_i$, $F(U)$ is $(i, j)$-preopen.

**Remark 5.1.** (a) By Remark 4.3(a), $(i, j)SO(Y)$, $(i, j)PO(Y)$, $(i, j)\alpha(Y)$ and $(i, j)SPO(Y)$ are all $m$-structures on $Y$ satisfying property ($\mathcal{B}$). Therefore, a function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(i, j)$-semi-open (resp. $(i, j)$-preopen, $(i, j)$-$\alpha$-open, $(i, j)$-semi-preopen) if and only if $f : (X, \tau_i) \rightarrow (Y, m_{ij})$ is $m$-open, where $m_{ij} = (i, j)SO(Y)$ (resp. $(i, j)PO(Y)$, $(i, j)\alpha(Y)$, $(i, j)SPO(Y)$).

(b) A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(i, j)$-preopen if $F : (X, \tau_i) \to (Y, m_{ij})$ is $m$-open, where $m_{ij} = (i, j)\mathrm{PO}(Y)$.

**Definition 5.3.** Let $(Y, \sigma_1, \sigma_2)$ be a bitopological space and $m_{ij} = m(\sigma_1, \sigma_2)$ an $m$-structure on $Y$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be *$(i, j)$-$m$-open* at $x \in X$ if $F : (X, \tau_i) \to (Y, m_{ij})$ is $m$-open at $x \in X$.

**Remark 5.2.** (a) A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is *$(i, j)$-$m$-open* at $x \in X$ if for each $\tau_i$-open set $U$ containing $x$, there exists $V \in m_{ij}$ containing $F(x)$ such that $V \subset F(U)$.

(b) By Remark 3.3, it follows from that a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(i, j)$-$m$-open, where $m_{ij} = m(\sigma_1, \sigma_2)$ has property $\mathcal{B}$, if and only if $F(U)$ is $m$-open for every $\tau_i$-open set $U$ of $X$.

(c) If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a function and $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction, then by Definiton 5.3 we obtain Definitions 5.1 and 5.2.

By Definition 5.3 and Theorems 3.1-3.5, we obtain the following theorems.

**Theorem 5.1.** *Let $(Y, \sigma_1, \sigma_2)$ be a bitopological space and $m_{ij} = m(\sigma_1, \sigma_2)$ an $m$-structure on $Y$ with property $\mathcal{B}$. Then a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(i, j)$-$m$-open at $x \in X$ if and only if $x \in F^+(m_{ij}\text{-Int}(F(U)))$ for every $\tau_i$-open set $U$ containing $x$.*

**Theorem 5.2.** *Let $(Y, \sigma_1, \sigma_2)$ be a bitopological space and $m_{ij} = m(\sigma_1, \sigma_2)$ an $m$-structure on $Y$ with property $\mathcal{B}$. Then a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(i, j)$-$m$-open if and only if $F(U)$ is $m_{ij}$-open for every $\tau_i$-open set $U$ of $X$.*

**Theorem 5.3.** *Let $(Y, \sigma_1, \sigma_2)$ be a bitopological space and $m_{ij} = m(\sigma_1, \sigma_2)$ an $m$-structure on $Y$ with property $\mathcal{B}$. For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *$F$ is $(i, j)$-$m$-open at $x \in X$;*

(2) *If $x \in i\mathrm{Int}(A)$ for $A \in \mathcal{P}(X)$, then $x \in F^+(m_{ij}\text{-Int}(F(A)))$;*

(3) *If $x \in i\mathrm{Int}(F^+(B))$ for $B \in \mathcal{P}(Y)$, then $x \in F^+(m_{ij}\text{-Int}(B))$;*

(4) *If $x \in F^-(m_{ij}\text{-Cl}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in i\mathrm{Cl}(F^-(B))$.*

**Theorem 5.4.** *Let $(Y, \sigma_1, \sigma_2)$ be a bitopological space and $m_{ij} = m(\sigma_1, \sigma_2)$ an $m$-structure on $Y$ with property $\mathcal{B}$. For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *$F$ is $(i, j)$-$m$-open;*

(2) *$F(i\mathrm{Int}(A)) \subset m_{ij}\text{-Int}(F(A))$ for every subset $A$ of $X$;*

(3) *$i\mathrm{Int}(F^+(B)) \subset F^+(m_{ij}\text{-Int}(B))$ for every subset $B$ of $Y$;*

(2) *$F(i\mathrm{Int}(A) \subset m_{ij}\text{-}\mathrm{Int}(F(A))$ for every subset $A$ of $X$;*

(3) *$i\mathrm{Int}(F^+(B)) \subset F^+(m_{ij}\text{-}\mathrm{Int}(B))$ for every subset $B$ of $Y$;*

(4) *If $F^-(m_{ij}\text{-}\mathrm{Cl}(B)) \subset i\mathrm{Cl}(F^-(B))$ for every subset $B$ of $Y$.*

For a function $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, we denote

$$D^0_{ij} = \{x \in X : F \text{ is not } (i, j)\text{-}m\text{-open at } x\},$$

then by Definition 5.3 and Theorem 3.5 we obtain the following theorem:

**Theorem 5.5.** *For a multifunction $F : (X, \tau) \to (Y, m_Y)$, where $m_{ij} = m(\sigma_1, \sigma_2)$ an m-structure on $Y$ with property $\mathcal{B}$, the following properties hold:*

$$\begin{aligned} D^0_{ij}(F) &= \cup_{U \in \tau_i} \{U - F^+(m_{ij}\text{-}\mathrm{Int}(F(U)))\} \\ &= \cup_{A \in \mathcal{P}(X)} \{i\mathrm{Int}(A) - F^+(m_{ij}\text{-}\mathrm{Int}(F(A)))\} \\ &= \cup_{B \in \mathcal{P}(Y)} \{i\mathrm{Int}(F^+(B)) - F^+(m_{ij}\text{-}\mathrm{Int}(B))\} \\ &= \cup_{B \in \mathcal{P}(Y)} \{F^-(m_{ij}\text{-}\mathrm{Cl}(B)) - i\mathrm{Cl}(F^-(B))\}. \end{aligned}$$

## 6. NEW FORMS OF MODIFICATIONS OF OPEN MULTIFUNCTIONS

There are many modifications of open sets in topological spaces. In order to define some new modifications of open sets in a bitopological space, let recall $\theta$-open sets and $\delta$-open sets due to Velicko [33]. Let $(X, \tau)$ be a topological space. A point $x \in X$ is called a $\theta$-cluster (resp. $\delta$-cluster) point of a subset $A$ of $X$ if $\mathrm{Cl}(V) \cap A \neq \emptyset$ (resp. $\mathrm{Int}(\mathrm{Cl}(V)) \cap A \neq \emptyset$) for every open set $V$ containing $x$. The set of all $\theta$-cluster (resp. $\delta$-cluster) points of $A$ is called the *$\theta$-closure* (resp. *$\delta$-closure*) of $A$ and is denoted by $\mathrm{Cl}_\theta(A)$ (resp. $\mathrm{Cl}_\delta(A)$). If $A = \mathrm{Cl}_\theta(A)$ (resp. $A = \mathrm{Cl}_\delta(A)$), then $A$ is said to be *$\theta$-closed* (resp. *$\delta$-closed*) [33]. The complement of a $\theta$-closed (resp. $\delta$-closed) set is said to be *$\theta$-open* (resp. *$\delta$-open*). The union of all $\theta$-open (resp. $\delta$-open) sets contained in $A$ is called the *$\theta$-interior* (resp. *$\delta$-interior*) of $A$ and is denoted by $\mathrm{Int}_\theta(A)$ (resp. $\mathrm{Int}_\delta(A)$).

**Definition 6.1.** A subset $A$ of a bitopological space $(Y, \sigma_1, \sigma_2)$ is said to be

(1) *$(i, j)$-$\delta$-semi-open* [27] if $A \subset j\mathrm{Cl}(i\mathrm{Int}_\delta(A))$, where $i \neq j$, $i, j = 1, 2$,

(2) *$(i, j)$-$\delta$-preopen* [28] if $A \subset i\mathrm{Int}(j\mathrm{Cl}_\delta(A))$, where $i \neq j$, $i, j = 1, 2$,

(3) *$(i, j)$-$\delta$-semi-preopen* (simply *$(i, j)$-$\delta$-sp-open*) if there exists an $(i, j)$-$\delta$-preopen set $U$ such that $U \subset A \subset j\mathrm{Cl}(U)$, where $i \neq j$, $i, j = 1, 2$.

**Definition 6.2.** A subset $A$ of a bitopological space $(Y, \sigma_1, \sigma_2)$ is said to be

(1) *$(i, j)$-$\theta$-semi-open* if $A \subset j\mathrm{Cl}(i\mathrm{Int}_\theta(A))$, where $i \neq j$, $i, j = 1, 2$,

(2) *$(i, j)$-$\theta$-preopen* if $A \subset i\mathrm{Int}(j\mathrm{Cl}_\theta(A))$, where $i \neq j$, $i, j = 1, 2$,

Let $(Y, \sigma_1, \sigma_2)$ be a bitopological space. The family of $(i, j)$-δ-semi-open (resp. $(i, j)$-δ-preopen, $(i, j)$-δ-*sp*-open, $(i, j)$-θ-semi-open, $(i, j)$-θ-preopen, $(i, j)$-θ-*sp*-open) sets of $(Y, \sigma_1, \sigma_2)$ is denoted by $(i, j)\delta SO(Y)$ (resp. $(i, j)\delta PO(Y)$, $(i, j)\delta SPO(Y)$, $(i, j)\theta SO(Y)$, $(i, j)\theta PO(Y)$, $(i, j)\theta SPO(Y)$).

**Remark 6.1.** Let $(Y, \sigma_1, \sigma_2)$ be a bitopological space. The family $(i, j)\delta SO(Y)$, $(i, j)\delta PO(Y)$, $(i, j)\delta SPO(Y)$, $(i, j)\theta SO(Y)$, $(i, j)\theta PO(Y)$ and $(i, j)\theta SPO(Y)$ are all *m*-structures with property $\mathscr{B}$.

For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, we can define many new types of $(i, j)$-*m*-open multifunctions. For example, in case $m_{ij} = (i, j)\delta SO(Y)$ (resp. $(i, j)\delta PO(Y)$, $(i, j)\delta SPO(Y)$, $(i, j)\theta SO(Y)$, $(i, j)\theta PO(Y)$, $(i, j)\theta SPO(Y)$) we can define new types of $(i, j)$-*m*-open multifunctions as follows:

**Definition 6.3.** A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be *$(i, j)$-δ-semi-open* (resp. *$(i, j)$-δ-preopen*, *$(i, j)$-δ-sp-open*) if $F : (X, \tau_i) \to (Y, m_{ij})$ is $(i, j)$-*m*-open and $m_{ij} = (i, j)\delta SO(Y)$ (resp. $(i,j)\delta PO(Y)$, $(i, j)\delta SPO(Y)$).

**Definition 6.4.** A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be *$(i, j)$-θ-semi-open* (resp. *$(i,j)$-θ-preopen*, *$(i, j)$-θ-sp-open*) if $F : (X, \tau_i) \to (Y, m_{ij})$ is $(i, j)$-*m*-open and $m_{ij} = (i, j)\theta SO(Y)$ (resp. $(i, j)\theta PO(Y)$, $(i, j)\theta SPO(Y)$).

**Conclusion.** We can apply the characterizations established in Sections 5 to the multifunctions defined in Definitions 6.3 and 6.4 and also to multifunctions defined by using any *m*-structure $m_{ij} = m(\sigma_1, \sigma_2)$ with property $\mathscr{B}$ determined by $\sigma_1$ and $\sigma_2$ in a bitopological space $(Y, \sigma_1, \sigma_2)$.

## REFERENCES


1. M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, *β-open sets and β-continuous mappings,* Bull. Fac. Sci. Assiut Univ. **12** (1983), 77-90.
2. M. E. Abd El-Monsef, R. A. Mahmoud and E. R. Lashin, *β-closure and β-interior,* J. Fac. Ed. Ain Shans Univ. **10** (1986), 235-245.
3. D. Andrijevic', *Semi-preopen sets,* Mat. Vesnik **38** (1986), 24-32.
4. D. Andrijevic', *On b-open sets,* Mat. Vesnik **48** (1996), 59-64.
5. T. Bânzaru, *Topologies on spaces of subsets and multivalued mappings,* Mathematical Monographs, University of Timişoara, 1997.
6. S. Bose, *Semi-open sets, semi-continuity and semi-open mappings in bitopological spaces,* Bull. Calcutta Math. Soc. **73** (1981), 237-246.
7. J. Cao and I. L. Reilly, *α-continuous and α-irresolute multifunctions,* Math. Bohemica **121** (1996), 415-424.

8. J. Cao and I. L. Reilly, *On pairwise almost continuous multifunctions and closed graph,* Indian J. Math. **38** (1996), 1-17.

9. S. G. Crossley and S. K. Hildeband, *Semi-closure,* Texas J. Sci. **22** (1971), 99-112.

10. N. El-Deeb, I. A. Hasanein, A. S. Mashhour and T. Noiri, *On p-regular spaces,* Bull. Math. Soc. Sci. Math. R. S. Roumanie **27(75)** (1983), 311-315.

11. A. A. El-Attik, *A Study of Type of Mappings in Topological Spaces,* M. Sci. Thesis, Tanta Univ. Egypt, 1997.

12. M. Jelić, *A decomposition of pairwise continuity,* J. Inst. Math. Comput. Sci. Math. Ser. **3** (1990), 25-29.

13. M. Jelić, *Feebly p-continuous mappings,* Suppl. Rend. Circ. Mat. Palermo (2) **24** (1990), 387-395.

14. M. Jelić, *On some mappings of bitopological spaces,* Suppl. Rend. Circ. Mat. Palermo (2) **29** (1992), 483-491.

15. A. Kar and P. Bhattacharyya, *Bitopological preopen sets, precontinuity and preopen mappings,* Indian J. Math. **34** (1992), 295-309.

16. F. H. Khedr, S. M. Al-Areefi and T. Noiri, *Precontinuity and semi-precontinuity in bitopological spaces,* Indian J. Pure Appl. Math. **23** (1992), 625-633.

17. S. Sampath Kumar, *Pairwise α-open, α-closed and α-irresolute functions in bitopological spaces,* Bull. Inst. Math. Acad. Sinica, **21** (1993), 59-72.

18. N. Levine, *Semi-open sets and semi-continuity in topological spaces,* Amer. Math. Monthly **70** (1963), 36-41.

19. T. Lipski, *Quasicontinuous multivalued maps in bitopological spaces,* Slupskie Prace Mat. Przyrodnicze Slupsk 7 (1988), 3-31.

20. S. N. Maheshwari and R. Prasad, *Semiopen sets and semi continuous functions in bitopological spaces,* Math. Notae **26** (1977/78), 29-37.

21. H. Maki, K. C. Rao and A. Nagoor Gani, *On generalizing semi-open and preopen sets,* Pure Appl. Math. Sci. **49** (1999), 17-29.

22. A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deep, *On precontinuous and weak precontinuous mappings,* Proc. Math. Phys. Soc. Egypt. **53** (1982), 47-53.

23. A. S. Mashhour, I. A. Hasanein and S. N. El-Deeb, *α-continuous and α-open mappings,* Acta Math. Hungar. **41** (1983), 213-218.

24. A. A. Nasef and T. Noiri, *Feebly open sets and feeble continuity in bitopological spaces,* An. Univ. Timişoara Ser. Mat.-Inform. **36** (1998), 79-88.

25. O. Njåstad, *On some classes of nearly open sets,* Pacific J. Math. **15** (1965), 961-970.

26. T. Noiri and B. Ahmad, *A note on semi-open functions*, Math. Sem. Notes Kobe Univ. **10** (1982), 437-441.

27. N. Palaniappan and S. Pinou Missier, *δ-semi-open sets in bitopological spaces,* J. Indian Acad. Math. **25** (2003), 193-207.

28. N. Palaniappan and S. Pinou Missier, *δ-preopen sets in bitopological spaces,* J. Indian Acad. Math. **25** (2003), 287-295.

29. V. Popa, Y. Kucuk and T. Noiri, *On upper and lower preirresolute multifunctions,* Pure Appl. Math. Sci. **44** (1997), 5-16.

30. V. Popa and T. Noiri, *On M-continuous functions,* Anal. Univ. "Dunarea de Jos" Galaţi, Ser. Mat. Fiz. Mec. Teor. (2) **18(23)** (2000), 31-41.

31. V. Popa and T. Noiri, *On the definiitons of some generalized forms of continuity under minimal conditions,* Mem, Fac. Sci. Kochi Univ. Ser. A Math. **22** (2001), 9-18.

32. V. Popa and T. Noiri, *A unified theory of weak continuity for functions,* Rend. Circ. Mat. Palermo (2) **51** (2002), 439-464.

33. N. V. Velicko. *H-closed topological spaces.* Amer. Math. Soc. Transl. **78** (1968). 103-108.

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