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Clean Armendariz Ring

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Abstract

The purpose of this paper is to introduce Clean Armendariz (Cl-Armendariz) rings which are a generalization of Armendariz rings. We investigate some kind of rings such as corner ring and Polynomial rings to see which ones are Cl-Armendariz.

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1 Introduction

Throughout this article, R denotes an associative ring with identity. For a ring R, $Cl(R)$, $Nil(R)$, $U(R)$, $M_n(R)$, $T_n(R)$, $Id(R)$, $C(R)$ and e_{ij} denote the set of Clean element, set of Nilpotent elements in R, set of Unit elements of R, the $n \times n$ matrix ring over R, the $n \times n$ upper triangular matrix ring over R, the set of idempotent elements of R, the center of R and the matrix with (i, j)-entry 1 and elsewhere 0, respectively.

A ring R is called Armendariz if whenever polynomials $f(x) = \sum_{i=0}^{m} a_i x^i$ and $g(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$ satisfy $f(x)g(x) = 0$ then $a_i b_j = 0$ for each i, j (The converse is always true). The study of Armendariz ring was initiated by Armendariz [1, Lemma1] and Rege and Chhawchharia used Nagata's method of Idealization to construct examples of both Armendariz rings and non-Armendariz rings in [12]. Some properties of Armendariz rings are given in [2].So far Armendariz rings are generalized in several forms [5]. Zhongkui et al., [11] called a ring R weak Armendariz if whenever polynomials $f(x) = a_0 + a_1 x + \cdots + a_m x^m$, $g(x) = b_0 + b_1 x + \cdots + b_n x^n \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_i b_j \in Nil(R)$ for all i and j. Razaghi and Sahebi [13] called a ring R is Idempotent Armendariz ring if whenever polynomials $f(x) = a_0 + a_1 x + \cdots + a_m x^m$, $g(x) = b_0 + b_1 x + \cdots + b_n x^n \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_i b_j \in Id(R)$ for all i and j. In this paper, we introduce Clean Armendariz (Cl-Armendariz) rings as a generalization of Armendariz rings.

2 Clean Armendariz Ring

A ring is called clean if for each element $x \in R$, $x = e + u$ such that $e \in Id(R)$ and $u \in U(R)$.

Definition 2.1. A ring R is said to be Clean Armendariz (Cl-Armendariz) ring if whenever polynomials $f(x) = \sum_{i=0}^{m} a_i x^i$ and $g(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$, satisfy $f(x)g(x) = 0$, then $a_i b_j \in Cl(R)$ for each i, j.

It is easy to see that every clean ring is Cl-Armendariz but the following example shows that the converse does not hold in general.

Example 2.2. Let $\mathbb{Z}$ be the ring of integers. Since $\mathbb{Z}$ is a reduced ring, it is Cl-Armendariz but $\mathbb{Z}$ is not clean ring.

Theorem 2.3. *Let R_α be a ring, for each $\alpha \in I$. Then any direct product of rings $\prod_{\alpha \in I} R_\alpha$ is Cl-Armendariz if and only if any R_α is Cl-Armendariz.*

Proof. Let R_α be Cl-Armendariz, for each $\alpha \in I$ and $R = \prod_{\alpha \in I} R_\alpha$. Let $f(x)g(x) = 0$ for some polynomials $f(x) = \sum_{i=0}^{m} a_i x^i$ and $g(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$ where $a_i = (a_{i_1}, a_{i_2}, \ldots, a_{i_\alpha}, \ldots)$, $b_j = (b_{j_1}, b_{j_2}, \ldots, b_{j_\alpha}, \ldots)$ are elements of the product ring R for each $1 \leq i \leq m$ and $1 \leq j \leq n$. Define $f_\alpha(x) = \sum_{i=0}^{m} a_{i_\alpha} x^i$, $g_\alpha(x) = \sum_{j=0}^{n} b_{j_\alpha} x^j \in R_\alpha[x]$ for any $\alpha \in I$. From $f(x)g(x) = 0$, we have $a_0 b_0 = 0, a_0 b_1 + a_1 b_0 = 0, \ldots, a_m b_n = 0$, and this implies

$$\begin{aligned} a_{0_1} b_{0_1} = a_{0_2} b_{0_2} = \cdots = a_{0_\alpha} b_{0_\alpha} = \cdots = 0 \\ a_{0_1} b_{1_1} + a_{1_1} b_{0_1} = a_{0_2} b_{1_2} + a_{1_2} b_{0_2} = \cdots = a_{0_\alpha} b_{1_\alpha} + a_{1_\alpha} b_{0_\alpha} = \cdots = 0 \\ a_{m_1} b_{n_1} = a_{m_2} b_{n_2} = \cdots = a_{m_\alpha} b_{n_\alpha} = \cdots = 0 \end{aligned}$$

This means that $f_\alpha(x) g_\alpha(x) = 0$ in $R_\alpha[x]$, for each $\alpha \in I$. Since R_α is Cl-Armendariz for each $\alpha \in I$, $a_{i_\alpha} b_{j_\alpha} \in Cl(R_\alpha)$. Now the equation $\prod_{\alpha \in I} Cl(R_\alpha) = Cl(\prod_{\alpha \in I} R_\alpha)$, implies that $a_i b_j \in Id(R)$, and so R is Cl-Armendariz. Conversely, assume that $R = \prod_{\alpha \in I} R_\alpha$ is Cl-Armendariz and $f_\alpha(x) g_\alpha(x) = 0$ for some polynomials $f_\alpha(x) = \sum_{i=0}^{m} a_{i_\alpha} x^i$, $g_\alpha(x) = \sum_{j=0}^{n} b_{j_\alpha} x^j \in R_\alpha[x]$, with $\alpha \in I$. Define $F(x) = \sum_{i=0}^{m} a_i x^i$, $G(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$, where $a_i = (0, \ldots, 0, a_{i_\alpha}, 0, \ldots)$, $b_j = (0, \cdots, 0, b_{j_\alpha}, 0, \ldots) \in R$. Since $f_\alpha(x) g_\alpha(x) = 0$, we have $F(x)G(x) = 0$. Since R is Cl-Armendariz, $a_i b_j \in Cl(R)$. Therefore $a_{i_\alpha} b_{j_\alpha} \in Cl(R_\alpha)$ and so R_α is Cl-Armendariz for each $\alpha \in I$. □

Corollary 2.4. *Let R be a ring. Then R is Cl-Armendariz if and only if $R[[x]]$ is Cl-Armendariz.*

Proof. Let R be a ring. We have

$$R[[x]] \cong \{(a_i) : a_i \in R, \text{ for all } i \geq 0\} = \prod_{i \geq 0} R.$$

Hence by this fact and Theorem 2.3, R is Cl-Armendariz if and only if $R[[x]]$ is Cl-Armendariz. □

It is clear that Armendariz ring is Cl-Armendariz but the following example shows that the converse does not hold in general.

Example 2.5. Let $\mathbb{Z}_3[x, y]$ be the polynomial ring over $\mathbb{Z}_3$ in commuting indeterminates x and y. Consider the ring $R = \mathbb{Z}_3[x, y]/(x^3, x^2y^2, y^3)$. The commutativity of R implies that it is Cl-Armendariz but R is not Armendariz ring by [10, Example 3.2].

Since $Nil(R) \subseteq Cl(R)$, every weak Armendariz ring is Cl-Armendariz. But the following example shows that every Cl-Armendariz is not weak-Armendariz.

Example 2.6. Let F be a Field, $R = M_2(F)$ and $R_1 = R[[t]]$. Consider the ring $S = \{\sum_{i=0}^{\infty} a_i t^i \in R_1 | a_0 \in kI \text{ for} k \in F\}$, where I is the identity Matrix. Since R is Clean, it is obvious that S is Cl-Armendariz. Now for $f(x) = e_{11}t - e_{12}tx$ and $g(x) = e_{21}t + e_{11}tx \in S[x]$, we have $f(x)g(x) = 0$, but $(e_{11}t)^2$ is not Nilpotent in S and so S is not weak Armendariz.

With the previous Example we can show that subring of Cl-Armendariz rings need not be Cl- Armendariz in general.

Example 2.7. Take S be the ring as in Example 2.6. We claim that $S[x]$ is not Cl-Armendariz. Let $f(y) = e_{11}tx - e_{12}txy$ and $g(y) = e_{21}tx + e_{11}txy$ be polynomials in $S[x][y]$. Then $f(y)g(y) = 0$. We show that $(e_{11}tx)^2$ is not in $Cl(S[x])$. Let $(e_{11}tx)^2 = r$. If $r \in Cl(S[x])$, then $r = e + u$ such that u is a unit in $S[x]$. Since S is commutative, it is Abelian and so by [7] $Id(R[x]) = Id(R)$. Therefore e must be in S. Thus $-e + r$ is a unit in $S[x]$ and elementary calculation shows that this is impossible.

We can see from the previous Example the polynomial ring over Cl-Armendariz rings is not Cl-Armendariz rings in general.

Theorem 2.8. *Let R be a ring. If $R[x]$ is Cl-Armendariz ring, then R is Cl-Armendariz. The converse holds when R is reduced.*

Proof. Suppose that $R[x]$ is Cl-Armendariz ring. Let $f(y) = \sum_{i=1}^{m} f_i(y)$ and $g(y) = \sum_{j=1}^{n} g_j(y)$ be nonzero polynomials in $R[y]$, such that $f(y)g(y) = 0$. Since $f_i g_j \in Cl(R[x])$ and $R \subseteq R[x]$, we have $a_i b_j \in R \cap Cl(R[x]) = Cl(R)$. Therefore, R is Cl-Armendariz. Conversely, Suppose that R be a Cl-Armendariz ring. Let $f(y) = \sum_{i=1}^{m} f_i(y)$, $g(y) = \sum_{j=1}^{n} g_j(y)$ be nonzero polynomials in $R[x][y]$ with $f(y)g(y) = 0$, where $f_i = a_{i_0} + a_{i_1}x + \cdots + a_{i_{v_i}}x^{v^i}$, $g_j = b_{j_0} + b_{j_1}x + \cdots + b_{j_{w_j}}x^{w^j} \in R[x]$ for each $1 \le i \le m$ and $1 \le j \le n$. Take a positive integer t that $t = deg(f_0) + deg(f_1) + \cdots + deg(f_m) + deg(g_0) + deg(g_1) + \cdots + deg(g_n)$ where the degree is as polynomials in x and the degree of zero polynomial is taken to be zero. Then $f(x^t) = f_0 + f_1x^t + \cdots + f_mx^{tm}, g(x^t) = g_0 + g_1x^t + \cdots + g_nx^{tn} \in R[x]$ and the set of coefficients of the $f_{i'}s$ (resp $g_{j'}s$) equals the set of coefficients of the $f(x^t)$ (resp ($g(x^t)$)). Since $f(y)g(y) = 0$, $f(x^t)g(x^t) = 0$. Since R is Cl-Armendariz, $(a_{i_{r_i}} b_{j_{s_j}}) \in Cl(R)$ where $0 \le r_i \le v_i$ and $0 \le s_j \le w_j$, and since R is reduced, $Cl(R) = Cl(R[x])$. So $f_i g_j \in Cl(R[x])$. It implies that $R[x]$ is Cl-Armendariz. □

Let R be a ring and $e \in Id(R)$. Then the two-sided pierce decomposition writes R as the direct sum of $eRe, eR(1-e), (1-e)Re$ and $(1-e)R(1-e)$.

Proposition 2.9. *Let R be a ring and $e \in Cl(R)$. Then the following statements are equivalent:*
(1) R is Cl-Armendariz.
(2) eRe and $(1-e)R(1-e)$ are Cl-Armendariz and R is an Abelian ring.

Proof. For convenience, we let $\bar{e} = 1 - e$. Suppose eRe and $\bar{e}R\bar{e}$ are Cl-Armendariz rings and R is Abelian. We use the pierce decomposition of the ring R and so

$$R \cong eRe \oplus eR\bar{e} \oplus \bar{e}Re \oplus \bar{e}R\bar{e}. \tag{1}$$

Now, since idempotents in R are central, $R \cong eRe \oplus \bar{e}R\bar{e}$ and so R is Cl-Armendariz ring by Theorem 2.3 . Conversely, Let R be Cl-Armendariz ring. Let $f(x) = \sum_{i=1}^{m} a_i x^i$, $g(x) = \sum_{j=1}^{n} b_j x^j \in (eRe)[x]$ such that $f(x)g(x) = 0$. Since R is Cl-Armendariz and $a_i b_j \in eRe \subseteq R$, then we have $a_i b_j \in Cl(R) \cap eRe = Cl(eRe)$. This means that eRe is Cl-Armendariz. Similarly we can show that $\bar{e}R\bar{e}$ is Cl-Armendariz ring. Now let e be an idempotent of R. Consider $f(x) = e - er(\bar{e})x$ and $g(x) = \bar{e} + er\bar{e}x$. Therefore $f(x)g(x) = 0$. By hypothesis $er\bar{e}$ is central and so $er\bar{e} = 0$. Hence $er = ere$ for each $r \in R$. Similarly consider $p(x) = \bar{e} - \bar{e}rex$ and $q(x) = e + \bar{e}rex$ in $R[x]$ for all $r \in R$. Then $p(x)q(x) = 0$. As before $\bar{e}re = 0$ and $ere = re$ for all $r \in R$. It follows that e is central element of R, that is, R is Abelian. □

Corollary 2.10. *Let R be a Cl-Armendariz ring, then so is e_iRe_i for each $e_i \in Id(R)$. The converse holds if $1 = e_1 + e_2 + \cdots + e_n$ where the $e_i^{,}s$, $1 \leq i \leq n$ are orthogonal central idempotents.*

Proof. We have $R \cong e_1Re_1 \oplus \cdots \oplus e_nRe_n$ and the proof is done. □

The following Example Shows every Abelian ring is not Cl-Armendariz.

Example 2.11. Let $R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{Z}) | a \equiv d(mod2), b \equiv c \cong 0(mod2) \right\}$. The only idempotents in R are $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, so R is an Abelian ring. Let $f(x) = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix} x$, $g(x) = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} x \in R[x]$. Then $f(x)g(x) = 0$, but $\begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$ is not Clean in R. Therefore R is not Cl-Armendariz.

A ring R is called right (left) principal projective (it or simply, right (left) p.p- ring) if the right (left) annihilator of an element of R is generated by an idempotent.

Theorem 2.12. *Let R be a ring. If R is Armendariz ring then R is Cl-Armendariz. The converse holds if R is a right (left) p.p.-ring.*

Proof. Let R be Cl-Armendariz ring and right p.p.ring. By Proposition 2.9, R is Abelian. Let $f(x) = \sum_{i=0}^{m} a_i x^i$ and $g(x) = \sum_{j=0}^{n} b_j x^j$ are polynomials in $R[x]$ such that $f(x)g(x) = 0$. Assume that $f(x)g(x) = 0$. So we have

$$a_0 b_0 = 0 \tag{2}$$

$$a_0b_1 + a_1b_0 = 0 \tag{3}$$

$$a_0b_2 + a_1b_1 + a_2b_0 = 0 \tag{4}$$

$$\cdots$$

By hypothesis there exist idempotents $e_i \in R$ such that $r(a_i) = e_iR$ for all i. So $b_0 = e_0b_0$ and $a_0e_0 = 0$. Multiplying (3) by e_0 from the right, we have $0 = a_0b_1e_0 + a_1b_0e_0 = a_0e_0b_1 + a_1b_0e_0 = a_1b_0$. By (3) $a_0b_1 = 0$ and so $b_1 = e_0b_1$. Again, multiplying (4) by e_0 from the right, we have $0 = a_0b_2e_0 + a_1b_1e_0 + a_2b_0e_0 = a_1b_1 + a_2b_0$. Multiplying this equation by e_1 from the right, we have $0 = a_1b_1e_1 + a_2b_0e_1 = a_2b_0$. Continuing this process, we have $a_ib_j = 0$ for all $1 \leq i \leq s$ and $1 \leq j \leq t$. Hence R is Armendariz. This completes the proof. □

The following Example shows that the assumption of "p.p.-ring" in Theorem 2.12 is necessary.

Example 2.13. Let $R = T(\mathbb{Z}_8, \mathbb{Z}_8)$. Then R be Cl-Armendariz ring. It is not Armendariz ring by [12, Example 3.2]. Moreover, since the principal ideal $I = \begin{pmatrix} 0 & \mathbb{Z}_8 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} R$ is not projective, R is not a right p.p.-ring.

Since the rings $M_n(R)$ and $T_n(R)$ contain non-central idempotents. Therefore they are not Abelian and so these rings are not Cl-Armendariz in general. Given a ring R and M a (R, S)- bimodule. The Nagata extension of R by M is the ring $T(R, M) = R \bigoplus M$ with the usual addition and the multiplication

$$(r_1, m_1)(r_2, m_2) = (r_1r_2, r_1m_2 + m_1r_2).$$

This is isomorphic to the ring of all matrices $\left(\begin{smallmatrix} r & m \\ 0 & r \end{smallmatrix}\right)$, where $r \in R$ and $m \in M$ and the usual matrix operations are used.

Proposition 2.14. *Let R and S be two rings and T be the triangular ring $T = \left(\begin{smallmatrix} R & M \\ 0 & S \end{smallmatrix}\right)$ (where M is an (R, S)-bimodule). Then the rings R and S are Cl- Armendariz if and only if T is Cl- Armendariz.*

Proof. Let R and S be Cl- Armendariz ring. Take ring $T = \left(\begin{smallmatrix} R & M \\ 0 & S \end{smallmatrix}\right)$ and $f(x) = \sum_{i=0}^{m} \left(\begin{smallmatrix} r_i & m_i \\ 0 & s_i \end{smallmatrix}\right)$, $g(x) = \sum_{j=0}^{n} \left(\begin{smallmatrix} r_j & m_j \\ 0 & s_j \end{smallmatrix}\right)$ satisfy $f(x)g(x) = 0$. Define

$$f_r(x) = \sum_{i=0}^{m} r_ix^i, g_r(x) = \sum_{j=0}^{n} r'_jx^j \in R[x] \tag{5}$$

and

$$f_s(x) = \sum_{i=0}^{m} s_ix^i, g_s(x) = \sum_{j=0}^{n} s'_jx^j \in S[x] \tag{6}$$

From $f(x)g(x) = 0$, we have $f_r(x)g_r(x) = f_s(x)g_s(x) = 0$ and since R and S are Cl-Armendariz ring, $r_ir'_j \in Cl(R)$ and $s_is'_j \in Cl(S)$ for each $1 \leq i \leq m$ and $1 \leq j \leq$

n. It is enough to prove that $\begin{pmatrix} r_ir'_j & r_im'_j+m_is'_j \\ 0 & s_is'_j \end{pmatrix} \in Cl(T)$. Let $a = r_ir'_j, b = s_is'_j, m = r_im'_j + m_is'_j$. We consider Matrix $\begin{pmatrix} a & m \\ 0 & b \end{pmatrix} \in \begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$. Since a, b are clean, $a = e + u$ such that $e = e^2, u \in U(R)$ and $b = f + v$ such that $f = f^2, v \in U(S)$. So $\begin{pmatrix} a & m \\ 0 & b \end{pmatrix} = \begin{pmatrix} e & 0 \\ 0 & f \end{pmatrix} + \begin{pmatrix} u & m \\ 0 & v \end{pmatrix}$such that$\begin{pmatrix} e & 0 \\ 0 & f \end{pmatrix}$is idempotent and$\begin{pmatrix} u & m \\ 0 & v \end{pmatrix}$ is unit in T. Conversely, let T be a Cl-Armendariz ring,

$$f_r(x) = \sum_{i=0}^{m} r_ix^i, g_s(x) = \sum_{j=0}^{n} r'_jx^j \in R[x] \tag{7}$$

such that $f_r(x)g_r(x) = 0$,

$$f_s(x) = \sum_{i=0}^{m} s_ix^i, g_s(x) = \sum_{j=0}^{n} s'_jx^j \in S[x] \tag{8}$$

such that $f_s(x)g_s(x) = 0$. If $f(x) = \sum_{i=0}^{m} \begin{pmatrix} r_i & m_i \\ 0 & s_i \end{pmatrix}$, $g(x) = \sum_{j=0}^{n} \begin{pmatrix} r'_j & m_j \\ 0 & s'_j \end{pmatrix} \in T[x]$ then $f_r(x)g_r(x) = 0$ and $f_s(x)g_s(x) = 0$ follow that $f(x)g(x) = 0$. Since T is a Cl-Armendariz ring, $\begin{pmatrix} r_i & 0 \\ 0 & s_i \end{pmatrix}\begin{pmatrix} r'_j & 0 \\ 0 & s'_j \end{pmatrix} = \begin{pmatrix} r_ir'_j & 0 \\ 0 & s_is'_j \end{pmatrix} \in Cl(T)$. This shows that R and S are Cl-Armendariz. □

Corollary 2.15. *The following are equivalent for a ring R.*

(1) A ring R is Cl- Armendariz;

(2) The trivial extension $T(R, R)$ of R is Cl- Armendariz;

(3) $T_n(R)$ is Cl- Armendariz for any $n \geq 2$;

(4) $R[x]/(x^n)$ is Cl- Armendariz where (x^n) is the ideal generated by x^n in $R[x]$.

Proposition 2.16. *Let R be a ring which 2 is invertible and $G = \{1, g\}$ be a group. Then RG is Cl -Armendariz if and only if R is Cl -Armendariz.*

Proof. Since 2 is invertible, we have $RG \cong R \times R$ via the map $\theta : a + bg \to (a + b, a - b)$. Then the result follows by Theorem 2.3. □

Proposition 2.17. *For a ring R suppose that R/I is Cl-Armendariz for some ideal I of R. If Idempotents lift modulo I and $I \subseteq J$. Then R is Cl-Armendariz.*

Proof. For convenience we let $\bar{r} = r + I$. Suppose that $f(x) = \sum_{i=0}^{m} a_ix^i$ and $g(x) = \sum_{j=0}^{n} b_jx^j$ are polynomials in $R[x]$ such that $f(x)g(x) = 0$. Then $(f(x)/I)(g(x)/I) = 0$. Since R/I is Cl-Armendariz, it follows that $\bar{a}_i\bar{b}_j \in Cl(R/I)$. So $\bar{a}_i\bar{b}_j = \bar{e} + \bar{u}$ such that $\bar{e} \in Id(R/I)$ and $\bar{u} \in U(R/I)$. Since every Idempotent lift modulo I and $I \subseteq J$, $a_ib_j \in Cl(R)$ for all i and j by proof of [4, Proposition6]. This complete the proof. □

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